

# MODAL ANALYSIS OF ARBITRARILY SHAPED IRISES IN WAVEGUIDES BY A HYBRID CONTOUR-INTEGRAL MODE-MATCHING METHOD

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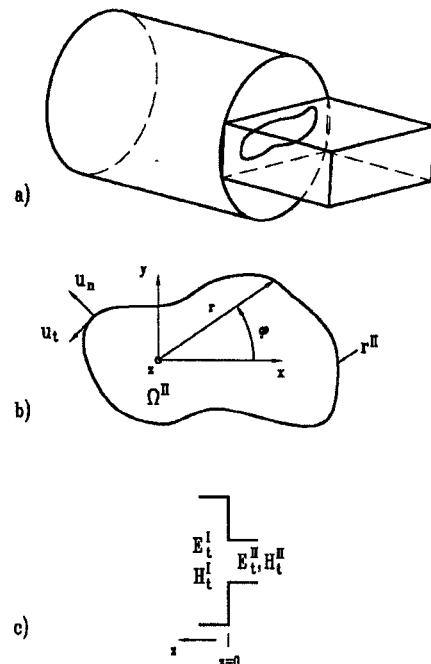
**Abstract** - A hybrid contour-integral mode-matching (CIMM) technique is presented for the rigorous calculation of the modal scattering parameters of arbitrarily shaped irises in waveguides which combines the advantages of flexibility and high numerical efficiency. The fast contour-integral matching yields the eigenmodes in the iris cross-sections for calculating the generalized scattering matrix of the complete iris discontinuity by the efficient mode-matching technique. This combined method takes arbitrarily iris geometries, arbitrary rotation of the iris, and the effects of finite thickness or different waveguide ports rigorously into account. The method is verified by comparison with results calculated by the finite element method.

## I. INTRODUCTION

ROGROUS CAD tools for waveguide components are of high importance for many design applications such as for space communication filters and multiplexers where accuracy, compactness and development time are very often the critical component design factors [1]. For structures composed of rectangular and circular waveguide segments, efficient mode-matching key-building blocks have already been developed which allow the rigorous formulation of single or multiple irises of rectangular or circular shape in rectangular or circular waveguides, [2] - [4].

For the analysis of more complicated structures, a boundary-contour mode matching method [6], a mixed mode matching finite difference method [7], a hybrid mode-matching finite element method [8], and a FD-TD method [9] have been proposed recently. However, although irises of more general shape, such as irises with rounded corners, have been already applied in many advanced filter designs for a rather long time, cf. [1], [5] (and the related literature cited there), no

rigorous simulation tools are available so far which allow their efficient direct CAD on usual workstations.



**Fig. 1:** Arbitrarily shaped iris with finite thickness between two different waveguides: (a) Iris structure. (b) General cross-section of the iris. (c) Step discontinuity.

The purpose of this paper is to present a very efficient contour-integral mode-matching (CIMM) technique for the rigorous calculation of the modal scattering parameters of arbitrarily shaped irises in rectangular and/or circular waveguides, Fig.1. The eigenmodes in the irises are determined by a contour-integral matching technique [10], and the generalized scattering matrix of the complete iris discontinuities are calculated by the well-proven mode-matching method. This hybrid method combines the flexibility of the contour-integral technique with the efficiency of the mode-matching method. Moreover, arbitrarily iris geometries, arbitrary rotation of the iris, and the effects of

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finite thickness or different waveguide ports are rigorously taken into account. The efficiency of the method is demonstrated at typical examples of high interest for filter and multiplexer designers: Rotated iris and rectangular iris with rounded corners in rectangular, circular and mixed rectangular/circular waveguides. The method is verified by comparison with results obtained by own calculations with the more time consuming finite element method.

## II. THEORY

For the cross-section (Fig. 1b) with the analytically or numerically given contour function  $r(\varphi)$  of the arbitrarily shaped iris (Fig. 1a), the fields are expanded in terms of the complete set of cylindrical wave functions

$$T(r, \varphi) = \sum_{n=0}^N J_n(k_c r) [a_n \cos(n\varphi) + b_n \sin(n\varphi)]. \quad (1)$$

The set of transversal eigenfunctions, equ. 1, is then multiplied with appropriate weighting functions  $\cos(j\varphi), \sin(j\varphi)$ , cf. [10], and integrated along the contour in order to satisfy the given field periodicity with respect to the angular coordinate  $\varphi$ . This relates the still unknown coefficients  $a_n, b_n$  in equ. 1 and the Fourier coefficients  $\alpha_j, \beta_j$  resulting from the contour integration, in the following manner

$$\begin{aligned} \alpha_j &= \sum_{n=0}^N \left[ \int_0^{2\pi} J_n(k_c r) a_n \cos(n\varphi) \cos(j\varphi) d\varphi \right. \\ &\quad \left. + \int_0^{2\pi} J_n(k_c r) b_n \sin(n\varphi) \cos(j\varphi) d\varphi \right], \end{aligned} \quad (2)$$

$$\begin{aligned} \beta_j &= \sum_{n=0}^N \left[ \int_0^{2\pi} J_n(k_c r) a_n \cos(n\varphi) \sin(j\varphi) d\varphi \right. \\ &\quad \left. + \int_0^{2\pi} J_n(k_c r) b_n \sin(n\varphi) \sin(j\varphi) d\varphi \right]. \end{aligned} \quad (3)$$

The requirement that the tangential electric field strength along the iris boundary contour is zero yields a homogeneous system of equations which may be written in matrix form

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \underbrace{\begin{bmatrix} [CC] & [SC] \\ [CS] & [SS] \end{bmatrix}}_{[C]} \underbrace{\begin{pmatrix} a \\ b \end{pmatrix}}_x = 0, \quad (4)$$

where the submatrices are denoted according to the  $\sin-$  and  $\cos-$ terms in equ. (2) - (3). The nontrivial solutions of this system of equations (4) result from  $\det[C] = 0$  which yields the eigenvalues, i.e. the cut-off wavenumbers  $k_c$ , of the iris waveguide section of arbitrary shape and finite length. The eigenvalues and the eigenvectors  $a, b$  are efficiently calculated by the singular value decomposition (SVD) method [11].

The modal scattering matrix of the discontinuity at  $z = 0$  (Fig. 1c) is obtained in the usual form [2] - [4], [6], by matching of the tangential field components. This yields the following equations where  $e, h$  are the transverse vector fields,  $Z, Y$  the wave impedances and admittances, and  $N$  the normalization expressions of the corresponding waveguide sections, respectively

$$\begin{aligned} E_t^I &= \sum_i e_i^{e,I} N_i^{e,I} Z_{0,i}^{e,I} (a_i^{e,I} + b_i^{e,I}) \\ &\quad + \sum_j e_j^{h,I} N_j^{h,I} (a_j^{h,I} + b_j^{h,I}) \Big|_{z=0}, \end{aligned} \quad (5)$$

$$\begin{aligned} E_t^{II} &= \sum_p e_p^{e,II} N_p^{e,II} Z_{0,p}^{e,II} (-a_p^{e,II} + b_p^{e,II}) \\ &\quad + \sum_q e_q^{h,II} N_q^{h,II} (a_q^{h,II} + b_q^{h,II}) \Big|_{z=0}, \end{aligned} \quad (6)$$

$$\begin{aligned} H_t^I &= \sum_i h_i^{e,I} N_i^{e,I} (a_i^{e,I} - b_i^{e,I}) \\ &\quad + \sum_j h_j^{h,I} N_j^{h,I} Y_{0,j}^{h,I} (a_j^{h,I} - b_j^{h,I}) \Big|_{z=0}, \end{aligned} \quad (7)$$

$$\begin{aligned} H_t^{II} &= \sum_p h_p^{e,II} N_p^{e,II} (a_p^{e,II} + b_p^{e,II}) \\ &\quad + \sum_q h_q^{h,II} N_q^{h,II} Y_{0,q}^{h,II} (-a_q^{h,II} + b_q^{h,II}) \Big|_{z=0}. \end{aligned} \quad (8)$$

Application of the orthogonality of the eigenfunctions and rearranging the equations yields the modal scattering matrix of the discontinuity directly in a similar manner as in [2] - [4], [6]. For the simplification of the calculation of the coupling integrals, all double integrals are reduced to contour integrals. The modal scattering matrix of cascaded structures (e.g. iris of finite thickness) is calculated by the known generalized modal S-matrix combination technique, [2] - [4], [6].

For all examples calculated in this paper, only fifteen eigenmodes in the iris cross-section structure and higher-order modes up to merely 90GHz cut-off frequency (in the subsequent order of increasing cut-off frequency) are required. The results are calculated by using a simple 486-level PC.

### III. RESULTS

Fig. 2 shows the scattering parameters (magnitude and phase) of a rectangular iris twisted by 45° between two rectangular waveguides calculated with our hybrid contour-integral mode-matching (CIMM) method. The iris thickness in all cases is assumed to be 0.21mm (i.e. appropriate for metal etching techniques). Excellent agreement with results obtained by an own MM/FE calculation, [8], may be observed which is chosen in order to verify the contour-integral mode-matching technique presented in this paper.

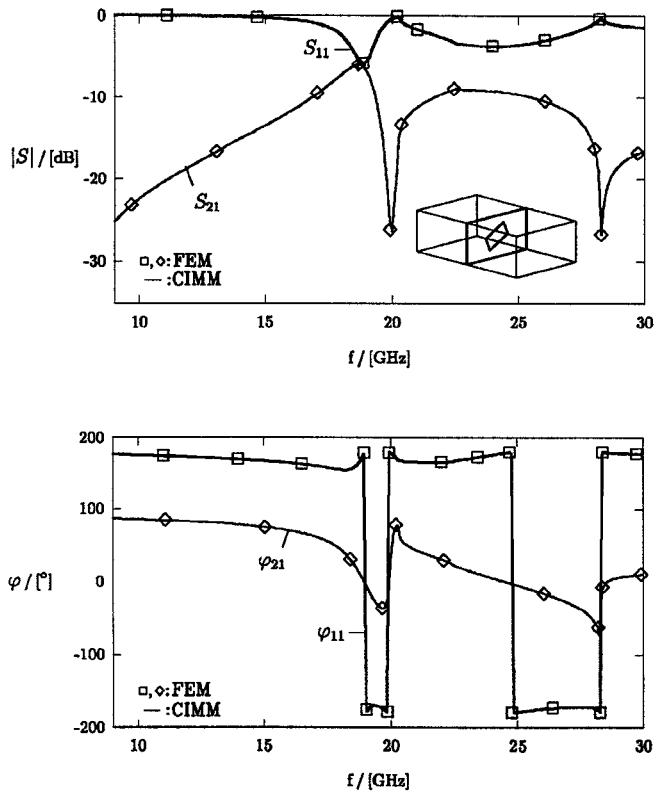


Fig. 2: Scattering parameters (magnitude and phase) of a rectangular iris ( $6 \times 2.4 \text{ mm}^2$ , thickness 0.21mm) rotated by 45° within two rectangular waveguide sections ( $20 \times 11 \text{ mm}^2$ ) of zero length. Comparison with results obtained by own calculations with the MM/FE method of [8].

Fig. 3 shows the results of a rectangular iris with rounded corners. Such irises play an important role in the fabrication of coupled cavity filters when typical milling tools are used which produce finite radii in the corners.

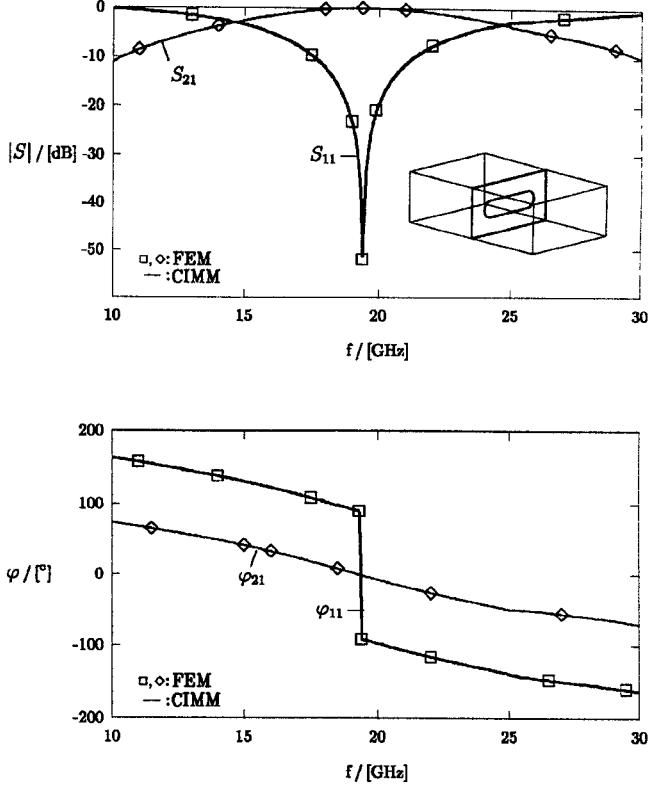


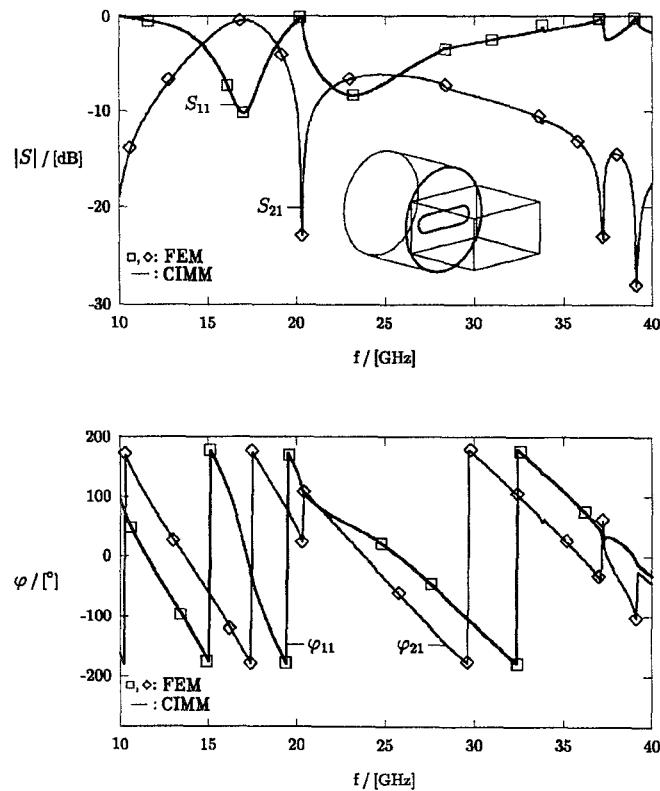
Fig. 3: Scattering parameters (magnitude and phase) of a rectangular iris with rounded corners ( $6.12 \times 3.4 \text{ mm}^2$ , radius 1.7mm, thickness 0.21mm) within two rectangular waveguide sections ( $18 \times 10 \text{ mm}^2$ ) of zero length. Comparison with results obtained by own calculations with the MM/FE method of [8].

In order to demonstrate the applicability of the presented method also for irises between different waveguides, Fig. 4 shows the scattering parameters of a rectangular iris with rounded corners between a circular and a standard rectangular WR 62-waveguide. Also for this case, excellent agreement with the results obtained by the more time consuming MM/FE method may be demonstrated.

### IV. CONCLUSION

An efficient field theory method is proposed for the rigorous modal scattering parameter analysis of arbitrarily shaped irises in rectangular, circular and mixed rectangular/circular waveguides. Such irises (of e.g. cross-sections with rounded corners) play an important role in modern space communication filter design. The method combines the flexibility of the contour-integral matching technique with the efficiency of the well-proven mode-matching method. Arbitrarily iris

geometries, arbitrary rotation of the iris, and the effects of finite thickness or different waveguide ports are rigorously taken into account. The method is verified by results obtained by the finite element method which requires more computational effort. Because of the high numerical efficiency of the proposed method, merely a standard PC is required for the rigorous analysis of arbitrarily shaped irises.



**Fig. 4:** Scattering parameters (magnitude and phase) of a rectangular iris with rounded corners ( $5.7 \times 3.0 \text{ mm}^2$ , radius  $1.5 \text{ mm}$ , thickness  $0.21 \text{ mm}$ ) between a rectangular  $WR62$ -waveguide ( $15.798 \times 7.899 \text{ mm}^2$ ) and a circular waveguide (radius  $9.0 \text{ mm}$ ) section of zero length. Comparison with results obtained by own calculations with the MM/FE method of [8].

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